Exponentials and Logarithms
Unit 9

	Section	Topic	Assignment
1	6.6	Natural Logarithms - Base *e*	**Pg 397** #40 -59 all
2	6.5	Exponential Equations	**Exponential Equations WS 1**
3	6.5	Logarithmic Equations	**Pg 382** #55 – 62 all **Logarithmic Equations WS 2**
4	6.1	Exponential Growth and Decay	**Exponential Growth and Decay WS 3**
5		Review	**Review WS**
6	Unit 9 Test		

Name_____

Natural Logarithms

Oral Exercises

Give each equation in exponential form.

1. $\ln 4 = 1.39$
2. $\ln \frac{1}{4} = -1.39$
3. $\ln e = 1$

Give each equation in logarithmic form.

4. $e^2 = 7.39$
5. $e^{-2} = 0.14$
6. $e^{1/5} = 1.22$

Simplify.

7. $\ln \frac{1}{e}$
8. $\ln e^{12}$
9. $\ln \sqrt{e}$

Written Exercises

Write each equation in exponential form.

A 1. $\ln 8 = 2.08$ 2. $\ln 100 = 4.61$ 3. $\ln \frac{1}{3} = -1.10$ 4. $\ln \frac{1}{e^2} = -2$

Write each equation in logarithmic form.

5. $e^3 = 20.1$
6. $e^7 = 1097$
7. $e^{1/2} = 1.65$
8. $\sqrt[3]{e} = 1.40$

Simplify. If the expression is undefined, say so.

9. $\ln e^2$
10. $\ln e^{10}$
11. $\ln \frac{1}{e^3}$
12. $\ln \frac{1}{\sqrt{e}}$
13. $\ln 1$
14. $\ln 0$
15. $e^{\ln 5}$
16. $e^{\ln 0.5}$

Write as a single logarithm.

17. $\ln 3 + \ln 4$
18. $\ln 8 - \ln 2$
19. $2 \ln 3 - \ln 5$
20. $\ln 7 + \frac{1}{2} \ln 9$
21. $\frac{1}{3} \ln 8 + \ln 5 + 3$
22. $4 \ln 2 - \ln 3 - 1$

Algebra 2

Name _____

Logarithmic Equations

Solve for x.

23. $\log_7 x = \log_7 4 + \log_7 3 - \log_7 2$ _____

24. $\log_3 x = 4 \log_3 2 + \log_3 5 - \log_3 4$ _____

25. $\log_5 x = \frac{1}{2}\log_5 9 + \log_5(x - 1)$ _____

26. $\log_6(x - 4) - \log_6 3 = \log_6 2$ _____

27. $\log_7 3x^2 = 2 \log_7 24 - \log_7 3$ _____

28. $2 \log_m(x + 1) - \log_m 4 = 0$ _____

Solve each equation.

33. $\log_a x = 2 \log_a 3 + \log_a 5$

34. $\log_a x = \frac{3}{2} \log_a 9 + \log_a 2$

35. $\log_b (x + 3) = \log_b 8 - \log_b 2$

36. $\log_b (x^2 + 7) = \frac{2}{3} \log_b 64$

37. $\log_a x - \log_a (x - 5) = \log_a 6$

38. $\log_a (3x + 5) - \log_a (x - 5) = \log_a 8$

39. $\log_2 (x^2 - 9) = 4$

40. $\log_3 (x + 2) + \log_3 6 = 3$

Kuta Software - Infinite Algebra 2 Name_____

Solving Exponential Equations with Logarithms Date_____ Period____

WS #1

Solve each equation. Round your answers to the nearest ten-thousandth.

1) $3^b = 17$

2) $12^r = 13$

3) $9^n = 49$

4) $16^v = 67$

5) $3^a = 69$

6) $6^r = 51$

7) $6^n = 99$

8) $20^r = 56$

9) $5 \cdot 18^{6x} = 26$

10) $e^{x-1} - 5 = 5$

11) $9^{n+10} + 3 = 81$

12) $11^{n-8} - 5 = 54$

13) $16^{n-7} + 5 = 24$

14) $20^{-6n} + 6 = 55$

15) $5 \cdot 6^{3m} = 20$

16) $8^{-5a} - 5 = 53$

17) $3.4e^{2-2n} - 9 = -4$

18) $-6e^{8n+8} - 3 = -23$

19) $-e^{-3.9n-1} - 1 = -3$

20) $-2e^{7v+5} - 10 = -17$

21) $-3e^{7a+9} + 6 = -6$

22) $-3e^{9x-1} + 6 = -58$

23) $-e^{6-9p} + 5 = -48.4$

24) $-10e^{2-2b} - 6 = -66$

25) $6e^{-4k-10} - 4 = 63$

26) $6e^{5x-6} - 4 = 50$

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Name_____

Logarithmic Equations

Date_____ Period____

WS #2

Solve each equation.

1) $\log 5x = \log (2x + 9)$

2) $\log (10 - 4x) = \log (10 - 3x)$

3) $\log (4p - 2) = \log (-5p + 5)$

4) $\log (4k - 5) = \log (2k - 1)$

5) $\log (-2a + 9) = \log (7 - 4a)$

6) $2\log_7 -2r = 0$

7) $-10 + \log_3 (n + 3) = -10$

8) $-2\log_5 7x = 2$

9) $\log -m + 2 = 4$

10) $-6\log_3 (x - 3) = -24$

11) $\log_{12} (v^2 + 35) = \log_{12} (-12v - 1)$

12) $\log_9 (-11x + 2) = \log_9 (x^2 + 30)$

13) $\log(16+2b) = \log(b^2-4b)$

14) $\ln(n^2+12) = \ln(-9n-2)$

15) $\log x + \log 8 = 2$

16) $\log x - \log 2 = 1$

17) $\log 2 + \log x = 1$

18) $\log x + \log 7 = \log 37$

19) $\log_8 2 + \log_8 4x^2 = 1$

20) $\log_9(x+6) - \log_9 x = \log_9 2$

21) $\log_6(x+1) - \log_6 x = \log_6 29$

22) $\log_5 6 + \log_5 2x^2 = \log_5 48$

23) $\ln 2 - \ln(3x+2) = 1$

24) $\ln(-3x-1) - \ln 7 = 2$

25) $\ln(x-3) - \ln(x-5) = \ln 5$

26) $\ln(4x+1) - \ln 3 = 5$

WS #3

Skill B Finding interest compounded continuously

Recall If the principal, P, is invested at an annual interest rate of r, compounded continuously, the amount, A, in the investment after t years is $A = Pe^{rt}$.

◆ **Example 1**
Find the amount in an account if \$1500 is invested at an annual rate of 5.8% and interest is compounded continuously for 7 years.

◆ **Solution**

$A = Pe^{rt}$
$A = 1500e^{(0.058)(7)}$ *5.8% = 0.058*
$A = \$2251.20$ *Use a calculator.*

◆ **Example 2**
How long will it take to double your money if you deposit \$500 at an annual rate of 7.2% compounded continuously?

◆ **Solution**

$A = Pe^{rt}$
$1000 = 500e^{(0.072)t}$ *\$500 doubles to \$1000*
$2 = e^{(0.072)t}$ *Divide each side by 500.*
$\ln 2 = \ln e^{(0.072)t}$ *Take the natural logarithm of each side.*
$\ln 2 = 0.072t$ *inverse functions: $\ln e^x = x$*
$t = \frac{\ln 2}{0.072}$
$t \approx 9.63$

It will take about 9 years and 7.5 months to double your money.

Find the amount, A, by using the formula $A = Pe^{rt}$ for continuously compounded interest.

13. \$1000 at 4% for 10 years

14. \$800 at 7% for 3 years

15. \$2500 at 5.2% for 6 years

16. \$10,000 at 12% for 10 years

17. \$8000 at 8.9% for 2 years

18. \$50,000 at 15% for 7 years

19. How long will it take to double your money if you deposit \$1200 at an annual rate of 6.9% compounded continuously? _____

20. How long will it take to triple your money if you deposit \$2000 at an annual rate of 8.5% compounded continuously? _____

21. **INVESTMENTS** Armando invests \$10,000 at an interest rate of 7.75%. If the interest is compounded continuously, and no additional deposits or withdrawals are made, how much is the account worth after 10 years? _____

22 \$1000 is deposited in an account with an interest rate of 6.5%. Interest is compounded continuously, and no deposits or withdrawals are made. Find the amount in the account at the end of three years. _____

23. The amount of radioactive carbon-14 remaining after t years is given by the formula $N(t) = N_0e^{-0.00012t}$. How much of a 20 milligram sample will remain after 50 years? _____

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Algebra 2

Name_____ _____

Exponential Growth and Decay

7. A certain population of bacteria doubles every 3 weeks. The number of bacteria in the population is now N_0. Find its size in:

a. 6 weeks **b.** 15 weeks **c.** W weeks

8. A culture of yeast doubles in size every 20 min. The size of the culture is now N_0. Find its size in:

a. 1 hour **b.** 12 hours **c.** 1 day

9. The half-life of carbon-14 is approximately 6000 years. Determine how much of 100 kg of this substance will remain after:

a. 12,000 years **b.** 24,000 years **c.** y years

10. The radioactive gas radon has a half-life of approximately $3\frac{1}{2}$ days. About how much of a 100 g sample will remain after 1 week?

Algebra II GHP
Unit 9 Test Review

Name_____
Date_____Period_____

I. Evaluate. **Round to the nearest hundredth.**

1) $\ln(4.7)$	2) $8.9e^{3.6}$

II. Write the equivalent logarithmic equation

3) $e^{-2.865} \approx 0.06$	4) $e^{-5x} = 6.78$
5) $e^{\frac{6}{x}} = 4.5$	6) $e^{7x-3} = 22$

III. Write the equivalent exponential equation

7) $\ln(7.3) \approx 1.99$	8) $\ln(86.4) = x$
9) $\ln\left(\frac{3}{x}\right) = 23.45$	10) $\ln(x) = 3.5$

IV. Simplify each expression.

11) $7.1 \ln e^3$	12) $e^{6 \ln 2}$

V. Use the Natural Logarithmic Properties to expand or express as a single natural logarithm.

13) **Expand.** $\ln\left(\frac{z^7 y}{x^{\frac{1}{2}}}\right)$	14) **Condense.** $2\ln(4p) - 5.28\ln(q) + \ln(r)$

VI. Solve for x. **Round to the nearest hundredth.**

15) $4(6)^x - 8 = 48$	16) $e^{2x+5} = 7$
17) $4^{\frac{3}{x}} = 11$	18) $9.3^x = 37$

VII. Solve for x. **Round to the nearest hundredth.**

19) $9.5e^{2.7x+9} = 28.5$	20) $4e^{-x-3} - 2 = 10$

VIII. Use Logarithmic Properties to solve the following equation for x.

21) $\log_5 (x-4)^8 = \log_5 7^8$
22) $\log_3 (x-8) = \log_3 2 + \log_3 8$
23) $\ln(6x) - \ln(5x+3) = 0$
24) $2\ln x + \ln 2 = \ln 2 + \ln(5x-6)$

IX. Find the amount A using the continuously compounded interest. **Round to the nearest hundredth.**

25) **What is the formula for continuously compounded interest?**_____

26) $1,250 at 5.6% for 5 years	27) $2,300 at 3.8% for 2 years

X. Find the time t using the continuously compounding interest. **Round to the nearest hundredth.**

28) How long will it take to double your money if you deposit $28,000 at an annual rate of 4.3% compounded continuously?
29) How long will it take to triple your money if you deposit $730 at an annual rate of 2.9% compounded continuously?

XI. **Round to the nearest hundredth.**

30) The amount of radioactive carbon-14 remaining after t years is given by the formula $N(t) = N_0e^{-0.0045t}$. How much of a 32 milligram sample will remain after 75 years?

Algebra 2
Unit 9

Name _____
Review WS

Applications of Logarithms; Exponential Growth and Decay; The Natural Logarithm Function

Use a calculator or Table 3 on text pages 812–813 to find each logarithm.

1. log 417 _____ 2. log 0.0611 _____ 3. log 909,000 _____

Use a calculator or Table 3 to find the number whose common logarithm is the value shown. Give the number to three significant digits.

4. $\log x = 1.8488$ _____ 5. $\log x = 4.2405$ _____

6. $\log x = 6.6920 - 10$ _____

Solve the equation first in calculation-ready form and then as a decimal with three significant digits.

7. $7^a = 39$ _____ 8. $11^m = 130$ _____ 9. $b^{2.7} = 57$ _____

10. $6^{t+1} = 19$ _____ 11. $7^{\frac{c}{3}} = 2.2$ _____ 12. $x = \log_3 10$ _____

Solve. Give the solution first in calculation-ready form and then as a decimal with three significant digits.

13. The half-life of the Sulfur-35 isotope is 87.1 days. How much of a 2 g sample will remain after 100 days? _____

14. Population studies of fiddler crabs on a tropical island reported 1.1×10^4 in 1980 and 1.5×10^4 in 1982. Predict the time that the maximum population, 2.0×10^4, that can be supported by the island's resources will be reached. _____

15. If Heather invests \$2000 in a fund that earns 10%, compounded every 6 months, how much will she have after 20 years? _____

Simplify.

16. $\ln e^5$ _____ 17. $\ln e^{\frac{1}{2}}$ _____ 18. $e^{\ln 3}$ _____

Write as a single logarithm.

19. $\ln 4 + \ln 3 - 6$ _____ 20. $\frac{1}{4} \ln 2 + \ln 4$ _____